Seat No. _______________

HA-003-2016002

B. Sc. (Sem. VI) (CBCS) (W.E.F. 2019) Examination

April - 2023

Mathematics : Paper - IX(A)

(Mathematical Analysis - II & Abstract Algebra - II)

Faculty Code : 003
Subject Code : 2016002

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Right hand side digit indicates the mark.

1 (a) Answer the following questions in short: **4**

(i) Disjoint subsets of a discrete metric space are separated. [True / False]

(ii) Subset of a connected set in metric space is connected if it is closed.
[True / False]

(iii) Define Disconnected set.

(iv) Define Homeomorphism.

(b) Attempt any one out of two: **2**

(i) Show that $\mathbb{Q}$ is not compact with discrete metric.

(ii) Prove or disprove "Every bounded set is totally bounded."

(c) Attempt any one out of two: **3**

(i) Let Y be a connected subset of a metric space (X, d). Then prove that Y can not be expressed as a disjoint union of two non-empty closed sets in Y.

(ii) If A is a connected subset of a metric space X and x is a limit point of A, then show that $A \cup \{x\}$ is also connected subset of a-metric space.

(d) Attempt any one out of two: **5**

(i) Prove that a metric space is a sequential compact if and only if it satisfies the Bolzano-Weirstrass theorem.

(ii) Show that every compact subset of a metric space is closed.

2 (a) Answer the following questions in short: **4**

(1) $L\left(e^{-at}\right) = \dfrac{1}{s+a}$, if ________.

(ii) $L^{-1}\left(\dfrac{1}{1+s^2}\right) =$ _______ .

(iii) Laplace transform is a linear transform with respect to the operation addition.
[True / False]

(iv) $L(5) =$ ______ .

(b) Attempt any one out of two: **2**

(i) Find $L^{-1}\left(\dfrac{as+b}{s^2+a^2}\right)$.

(ii) Prove that $L\{a f(t) + bg(t)\} = aL\{f(t)\} + bL\{g(t)\}$, where $a, b \in \mathbb{R}$.

(c) Attempt any one out of two: **3**

(i) Evaluate $L\{\cos(t)\cos(2t)\cos(3t)\}$.

(ii) State and prove first shifting theorem.

(d) Attempt any one out of two: **5**

(i) $L^{-1}\left(\dfrac{s}{s^4+4a^4}\right)$

(ii) Find the Laplace transform of $f(t)=|t-1|+|t-2|$; $t \ge 0$.

3 (a) Answer the following questions in short: **4**

(i) $L\left\{\dfrac{f(t)}{t}\right\}=$ _______ .

(ii) $L\{t\sin(t)\}=$_______ .

(iii) Convolution product is commutative.
[True / False]

(iv) Define Convolution product.

(b) Attempt any one out of two: **2**

(i) Find $L\left\{t\cdot e^{2t}\cdot\cos 3t\right\}$.

(ii) If $L\{f(t)\}=\overline{f}(s)$, then prove that

$$L\left\{t^n f(t)\right\}=(-1)^n\frac{d^n}{ds^n}\left[\overline{f}(s)\right],\ \text{for } n\in\mathbb{N}.$$

(c) Attempt any one out of two: **3**

(i) Find $L^{-1}\left(\dfrac{1}{(s-1)\left(s^2+1\right)}\right)$.

(ii) Find $L\left(\dfrac{\cos(2t)-\cos(3t)}{t}\right)$.

(d) Attempt any one out of two: **5**

(i) Solve $\left(D^3 - 3D^2 + 3D - 1\right) y = t^2 e^t$ given that $y(0) = 1,\ \ y'(0) = 0, y''(0) = -2$.

(ii) If $L\{f(t)\} = \overline{f}(s)$, then prove that

$L\left[\int_0^t f(u)\,du\right] = \frac{\overline{f}(s)}{s}$ and using that find

$L^{-1}\left(\frac{1}{s(s+a)^3}\right)$.

4 (a) Answer the following questions in short: **4**

(i) Write all zero divisor of $\left(\mathbb{Z}_{12}, +_{12}, \cdot_{12}\right)$.

(ii) Give an example of ring which is not an integral domain.

(iii) Define Field.

(iv) Define Homomorphisum of groups.

(b) Attempt any one out of two: **2**

(i) Prove that a field has no proper ideal.

(ii) If $\phi : (G, *) \to (G', \oplus)$ is a homomorphisum and H is a sub group of G, then show that $\phi(H)$ is a sub group of G'.

(c) Attempt any one out of two: **3**

(i) Show that the characteristic of an integral domain is either a prime number or zero.

(ii) For non zero polynomials $f(x)$ and $g(x)$ in $F(x)$, show that $f(x)g(x)$ is also non zero.

(d) Attempt any one out of two: **5**

(i) If $x=(0,1,0,......)\in D[x]$ then show that $x^n=(a_0,a_1,a_2,.....)$ for each positive integer n with $a_n=1$ and $a_i=0$ for each non-negative integer $i\neq n$.

(ii) State and prove first fundamental theorem of Homomorphisum.

5 (a) Answer the following questions in short : **4**

(i) Find all zeros of x^2-5x+6 in $\mathbb{Z}_{12}$.

(ii) Define monic polynomial.

(iii) Define irreducible polynomial.

(iv) State Unique Factorization Theorem.

(b) Attempt any one out of two: **2**

(i) For $f(x),g(x),h(x)\in F[x]$, if $f(x)\big|\big(h(x)g(x)\big)$ and *gcd* of $f(x)$ and $g(x)$ is 1, then prove that $f(x)\big|h(x)$.

(ii) Show that the polynomial x^2+1 is irreducible as an element of $\mathbb{Q}[x]$ but reducible as an element of $\mathbb{Z}_5[x]$.

(c) Attempt any one out of two: **3**

(i) For associate polynomials $f(x),g(x)\in F[x]$, prove that $f(x)=cg(x)$ for some $c\neq 0$ in *F*.

(ii) Suppose $f(x)\in F[x]$ with $[f(x)]=n$. If c is a leading coefficient of $f(x)$ and if $f(x)$ has n distinct zeros $a_1,a_2,....,a_n$ in *F*, then show that $f(x)=c(x-a_1)(x-a_2)........(x-a_n)$.

(d) Attempt any one out of two: **5**

(i) Using the Euclid's algorithm for

$f(x) = x^5 + 3x^3 + x^2 + 2x + 2 \in \mathbb{Z}_5[x]$ and

$g(x) = x^4 + 3x^3 + 2x^2 + x + 2 \in \mathbb{Z}_5[x]$, find *gcd* of

$f(x)$ and $g(x)$. Also express their *gcd* into the form

$a(x) f(x) + b(x) g(x)$.

(ii) State and prove division algorithm for polynomials in $F[x]$.
